Introduction to Computational Cognitive Science
AS.050.202 (Fall 2019)

Lab 3
(09/13/19)

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What is probability?

Proportion of times a particular event occurs relative to all possible events

$$P(E = x) = \frac{count(E = x)}{\sum_{y \in S} count(E = y)}$$

E: Random variable. Stores the value of the outcome of an event

S: Sample space (or event space). Set of all of all possible events (the range of values E can take)

Examples of sample spaces

- Coin: {H, T}
- Dice: {1, 2, 3, 4, 5, 6}
- Letters: {A, B, C … Z}
- Weather: {Hot, Cold, Rainy}
- Categories: {Dax, Bloop}

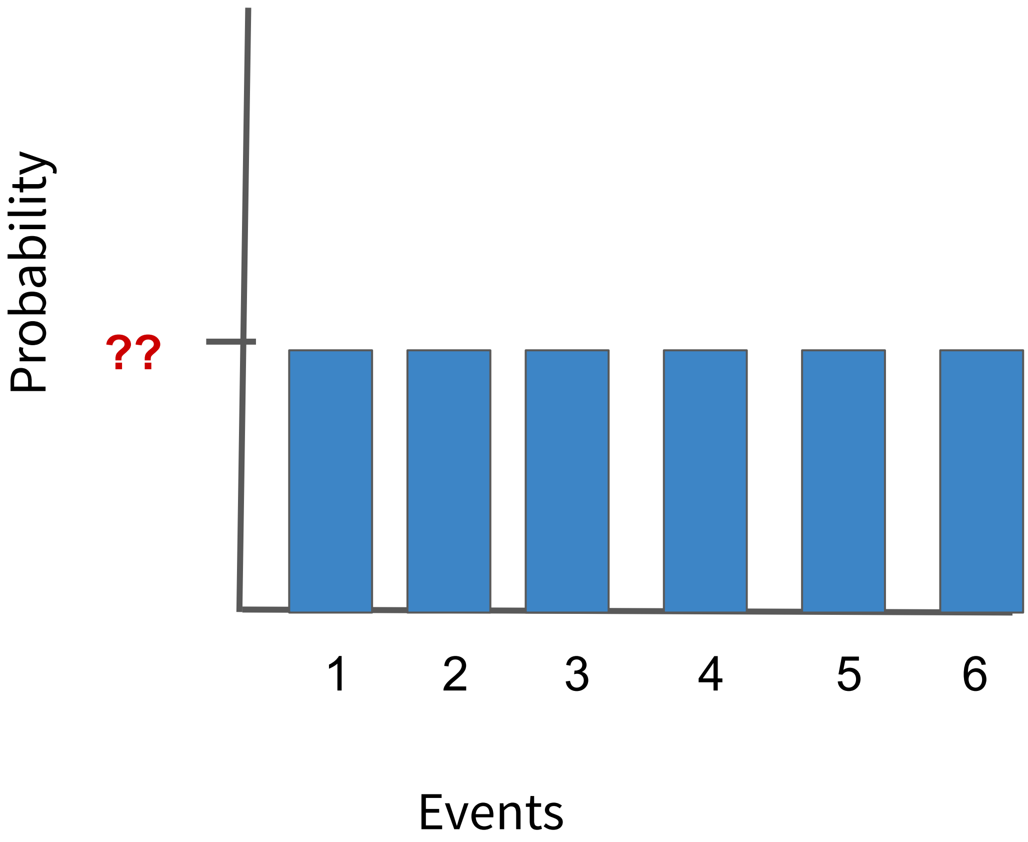
Discrete

- Height: {0 … 0.001 … 0.1 … 1.001 … 10.00001 ….}
- Temperature: {-273.15 …. $55*10^{11}$}

Continuous

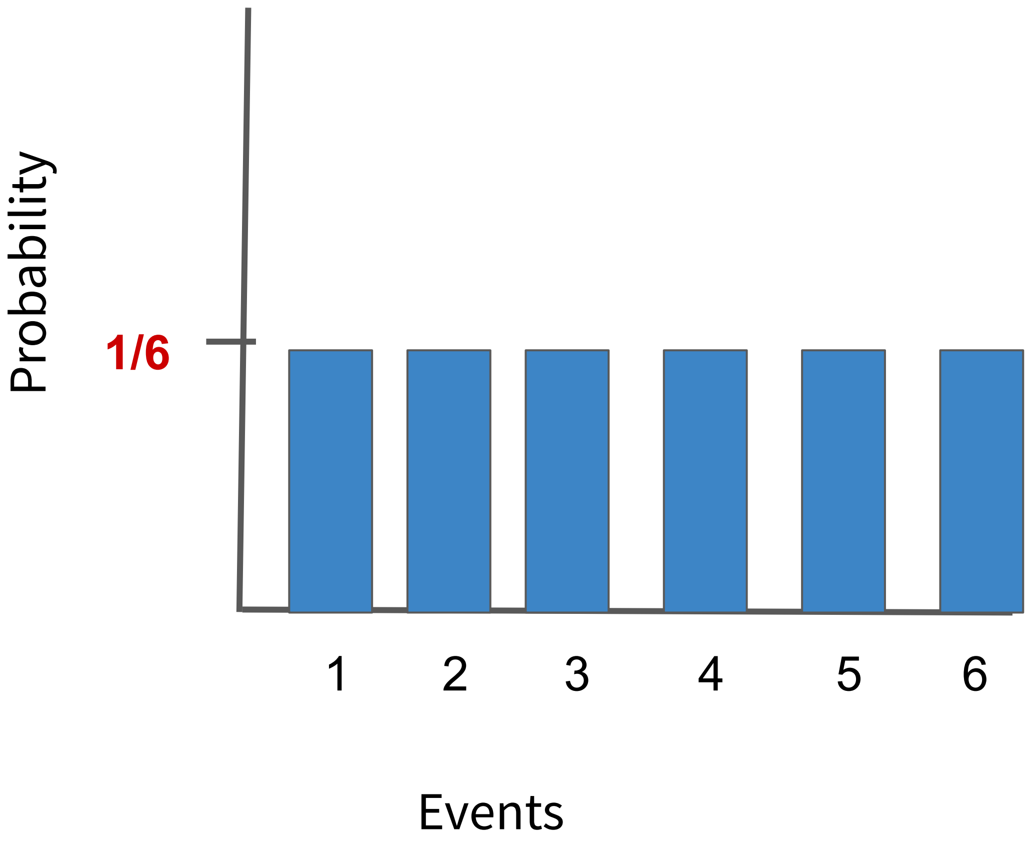
Probability mass (discrete events)

For each event, the probability that it occurs



Probability mass (discrete events)

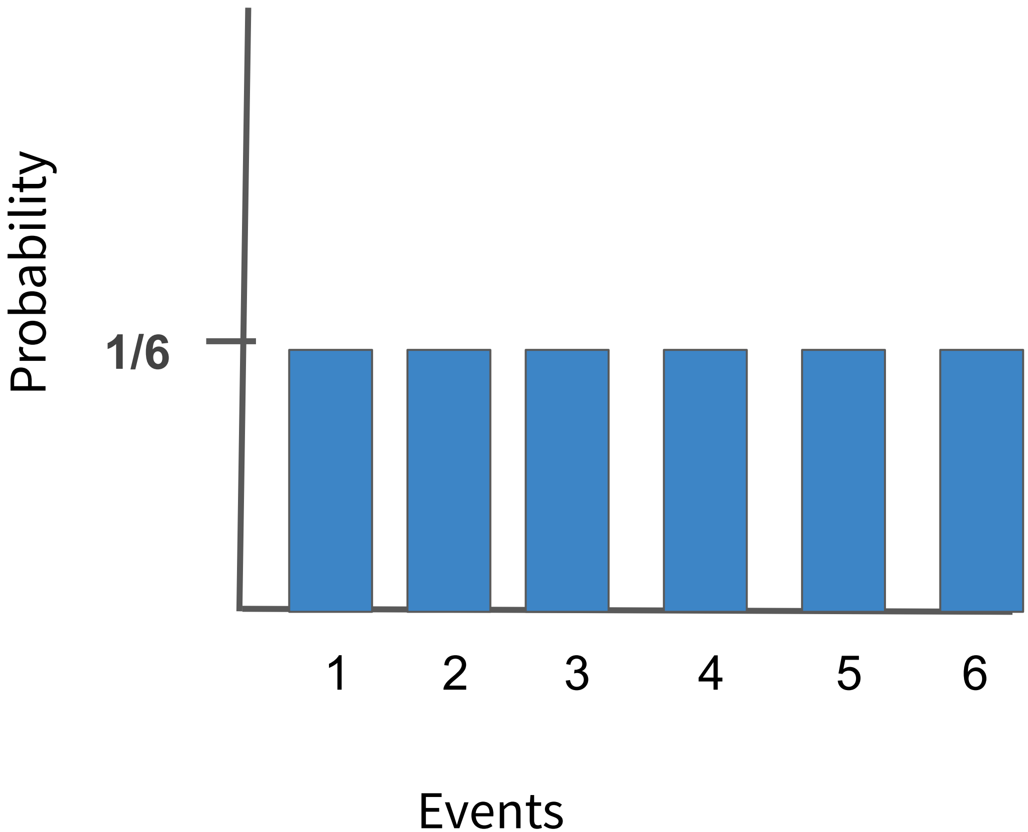
For each event, the probability that it occurs



The probability mass for all possible events must sum to 1

Probability mass (discrete events)

For each event, the probability that it occurs



P(E = 1)

P(E = 1 or E = 3)

P(E = odd)

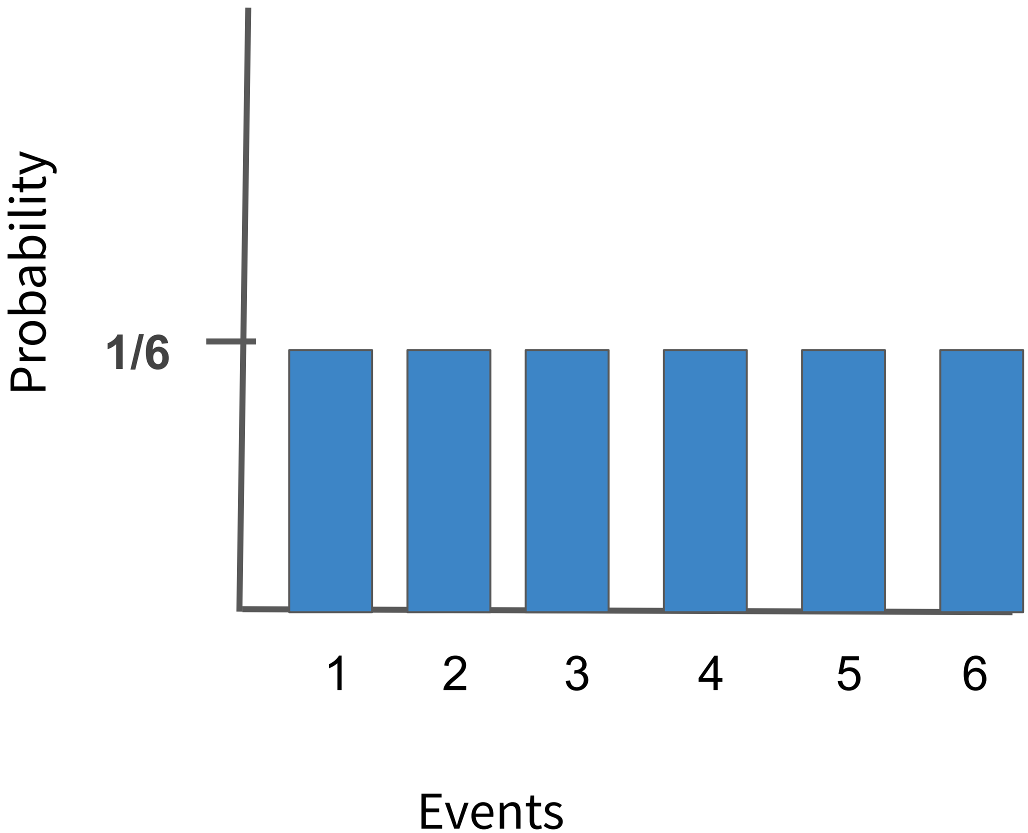
P(E > 6)

P(E = 1 and E = 3)

P(E >= 1)

Probability mass (discrete events)

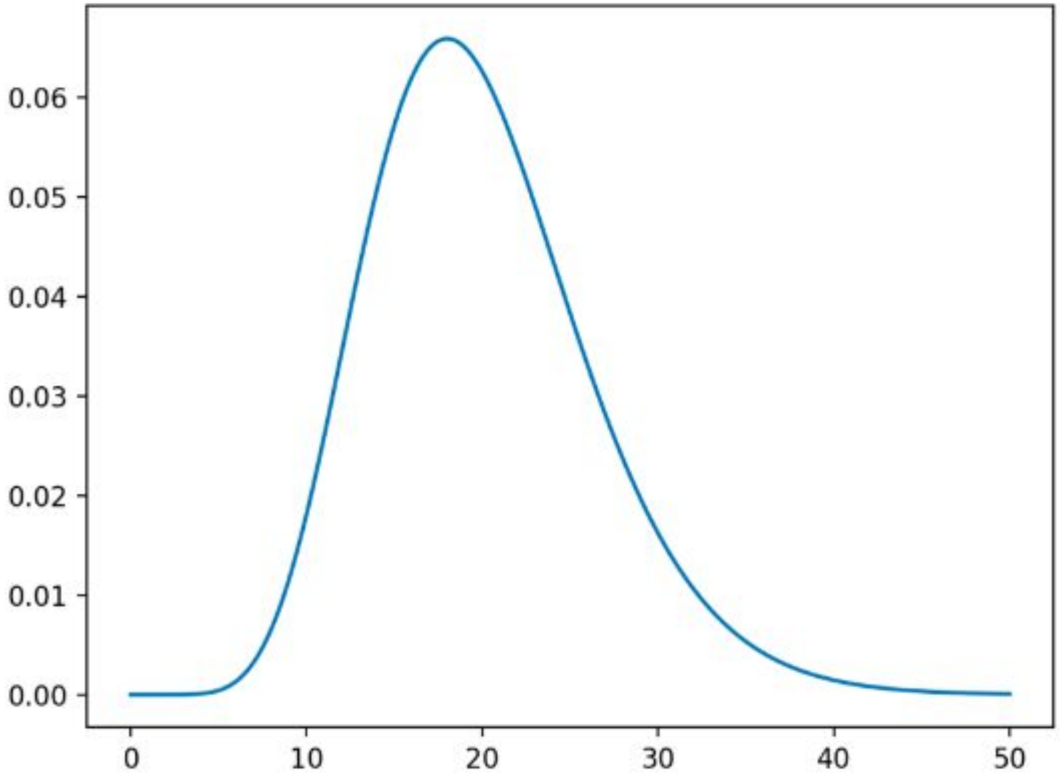
For each event, the probability that it occurs



P(E = 1)	1/6
P(E = 1 or E = 3)	1/3
P(E = odd)	1/2
P(E > 6)	0
P(E = 1 and E = 3)	0
P(E >= 1)	1

Probability density (continuous events)

Like probability mass, but the probability at any given point (e.g. event = 20.0000000000) will be zero. Take the area under the curve



Area under the curve = 1

Multiple events

If we roll two fair dice …

- What is the sample space of possible outcomes?

- What would the probability mass function look like?

- What is the probability that one of the dice lands on 1 and the other lands on 5?

Multiple events

If we roll two fair dice …

- What is the sample space of possible outcomes?

  { (1,1), (1,2) … (2,1), (2,2) … (6,6) }

- What would the probability mass function look like?

  Uniform distribution, with probability mass = 1/36

- What is the probability that one of the dice lands on 1 and the other lands on 5?

  P(1,5) + P(5,1) = 2/36

Multiple events

What are some other kinds of sample spaces we can have?

If we roll two fair dice …

- What is the **sample space of possible outcomes?**

  { (1,1), (1,2) … (2,1), (2,2) … (6,6) }

- What would the probability mass function look like?

  Uniform distribution, with probability mass = 1/36

- What is the probability that one of the dice lands on 1 and the other lands on 5?

  P(1,5) + P(5,1) = 2/36

Multiple events

If we roll two fair dice and take the **sum** of both rolls:

- What is the sample space of possible sums?
- What would the probability mass function look like?
- What is the probability of sum > 5 (how would you calculate it from your PMF?)
- What is the probability that sum is odd? (how would you calculate it from your PMF?)
- What is the probability that the sum is **not** 5?

Tutorial: https://tommccoy1.github.io/techviz/bayes/dice.html

Multiple events

If we roll two fair dice and take the **sum** of both rolls:

- What is the sample space of possible sums? **{2, 3, 4, … 12}**
- What would the probability mass function look like?
- What is the probability of sum > 5 (how would you calculate it from your PMF?) **26/36**
- What is the probability that sum is odd? (how would you calculate it from your PMF?) **1/2**
- What is the probability that the sum is **not** 5?

  1 - (4/36) = 32/36

Tutorial: https://tommccoy1.github.io/techviz/bayes/dice.html

Joint probability

P(A, B) is the probability of both A and B occurring.

- P(sum = 6, contains 2) =
- P(sum > 5, second roll is odd) =
- P(first roll is 1, second roll < 3) =

Joint probability

P(A, B) is the probability of both A and B occurring.

- P(sum = 6, contains 2) = **2/36**
- P(sum > 5, second roll is odd) = **12/36**
- P(first roll is 1, second roll < 3) = **2/36**

Joint probability and conditional probability

P(A, B) = P(A) * P(B | A) = P(B) * P(A | B)

Conditional probability:

P(B) given that A is true.

P(A) given that B is true

Conditionally independence:

- P(A | B) = P(A)
- P(B | A) = P(B)

When A and B are conditionally independent:

P(A, B) = **??**

Joint probability and conditional probability

P(A, B) = P(A) * P(B | A) = P(B) * P(A | B)

Conditional probability:

P(B) given that A is true.

P(A) given that B is true

Conditionally independence:

- P(A | B) = P(A)
- P(B | A) = P(B)

When A and B are mutually exclusive/ conditionally independent:

P(A, B) = **P(A) * P(B)**

Joint probability

P(A, B) is the probability of both A and B occurring.

- P(sum = 6, contains 2) = **2/36**
- P(sum > 5, second roll is odd) = **12/36**
- P(first roll is 1, second roll < 3) = **2/ 36**

Verify these answers using the formula

$$P(A, B) = P(A) * P(B \mid A) = P(B) * P(A \mid B)$$

Joint probability

P(A, B) is the probability of both A and B occurring.

- P(sum = 6, contains 2) = **2/36**
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Verify these answers using the formula

$$P(A, B) = P(A) * P(B \mid A) = P(B) * P(A \mid B)$$

Reminder:

Probability of two events **not** occurring together is 1 - P(A, B)

Product rule/ Chain rule

Remember: $P(A, B) = P(B) * P(A \mid B)$

$$P(X, (Y, Z)) = P(Y,Z) * P(X \mid Y, Z) \qquad (1)$$

$$P(Y,Z) = P(Z) * P(Y \mid Z) \qquad (2)$$

Combining (1) and (2)

$$P(X, Y, Z) = P(Z) * P(Y \mid Z) * P(X \mid Y, Z)$$

Bayes rule

$P(A, B) = P(A) * P(B \mid A)$

$P(A,B) = P(B) * P(A \mid B)$

$\Rightarrow P(A) * P(B \mid A) = P(B) * P(A \mid B)$

$$\Rightarrow P(B \mid A) = \frac{P(B) * P(A \mid B)}{P(A)}$$